Quest for Mathematics I (E2): Exercise sheet 2

1. Giving your argument,

   (a) evaluate $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$;

   (b) evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$;

   (c) express $0.343434\ldots$ as a fraction.

2. Do the following series converge or diverge (you should justify your answer):

   (a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ (where x is any real number);

   (b) $\sum_{n=1}^{\infty} \frac{2^n}{n^5}$;

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. *Hint: Consider pairing terms.*

3. Consider the following series:

$$x + x(1-x)^2 + x(1-x)^4 + x(1-x)^6 + \ldots.$$

   (a) Determine the interval I on which the series converges.

   (b) For $x \in I$, evaluate the limit, $f(x)$ say, and plot this as a graph.

   (c) For $x \in I$, state whether the function f is: continuous; discontinuous, but admits left or right limits; or is discontinuous in some other way.

4. The function

$$f(x) = \frac{x^3 - x^2}{x^3 - x}$$

   is well-defined and continuous wherever $x^3 - x \neq 0$. For points where $x^3 - x = 0$, deduce the value that should be assigned to f at x to ensure the function is continuous there, or explain why there is no such value.

5. Identify the point(s) of discontinuity of the following function:

$$f(x) = \left\lfloor \frac{1-x^2}{1+x^2} \right\rfloor.$$

   For each of the point(s), briefly describe the nature of the discontinuity (e.g. jump type, removable, asymptotic, etc).